

VACUUM PRELOADING WITH VERTICAL DRAINS THEORY AND RECENT DEVELOPMENTS - APPLICATIONS

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1. Introduction

Vacuum induced Consolidation (VIC), firstly proposed by Kjellman in 1952, is an effective means for accelerating the improvement of saturated soft soils while preventing pre-loading induced formation failures. The soil site is covered with an airtight membrane and a vacuum state is created underneath it by using vacuum pumps. The technology can provide an equivalent pre-loading of about 4.5m high as composed with a conventional surcharging fill. Instead of increasing the effective stress in the soil mass by increasing the total stress conventional mechanical surcharging, vacuum-assisted consolidation preloads the soil by reducing the pore pressure while maintaining a constant total stress. A typical construction site layout of vacuum preloading is showed in Fig 1.



Fig 1. Site layout of vacuum preloading (a site in Nanjing, China)

A nominal vacuum load of 80 kPa is normally used in design although a higher vacuum pressure of up to 90 kPa may be achieved sometimes. When a surcharge load higher than 80 kPa is required, a combined vacuum and fill surcharge can be applied. For the treatment of very soft ground, the vacuum preloading method is faster than the fill surcharge method, as the 80 kPa vacuum pressure can be applied almost instantly, without causing stability problem. Field experience indicates a substantial cost and time savings by this technology compared to conventional surcharging. The vacuum preloading method is cheaper, about 30%, compared with the fill surcharge method for an equivalent load (Shang et al. 1998, Chu et al. 2000).

This method has been successfully used for soil improvement or land reclamation projects in a number of countries like China, France, Japan, Thailand, Singapore (Chen and Bao 1983; Bergado et al. 1998; Chu et al. 2000; Indraratna et al. 2005a). Recently, the steadily increasing direct and indirect costs of placing and removing surcharge fill and the advent of technology for sealing landfills with impervious membranes for landfill gas extraction systems have now made vacuum-consolidation an economically viable method as a replacement for or supplement to surcharge fill.

An overview on techniques, current theory and practical aspects of vacuum preloading with vertical drains, some recent developments in vacuum preloading techniques...are briefly summarized in this paper.

2. Characteristics of vertical drain systems

2.1. Purpose and Application

Various types of vertical drains, including sand drains, sand compaction piles, prefabricated vertical drains (PVDs) and gravel piles have commonly been used in the past. Apart from the increasing cost of sand quarrying and strict environmental regulations in many countries, plus conventional sand drains that can be adversely affected by lateral soil movement, flexible PVD systems enjoying relatively rapid installation, have often replaced

the original sand drains and gravel piles. The most common band shaped (wick) drains have cross sectional dimensions of 100mm x 4mm. For design purposes the rectangular (width-a, thickness-b) section must be converted to an equivalent diameter, d_w , because the conventional theory of radial consolidation assumes circular drains (Fig 2).

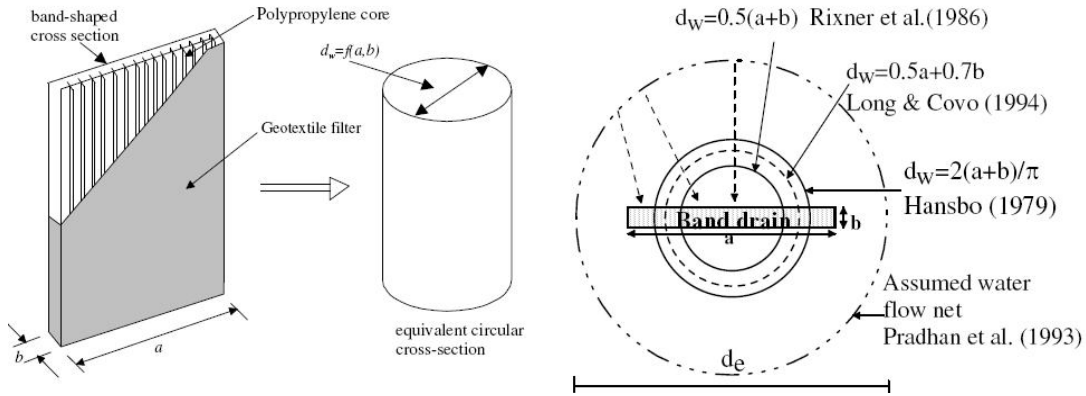


Fig 2. Conceptual illustration and assessment of equivalent diameter of band-shaped PVD (after Idraratna et al, 2005)

Fig 2 defines the appropriate dimensions of the band shaped PVD and the equivalent transformation to a circular cross section. The following typical equation based on a circular perimeter is used to determine the equivalent drain diameter (Hansbo, 1979):

$$d_w = 2(a + b) / \pi \quad (1)$$

Rixner et al. (1986) indicated that, due to the corner effect, the equivalent drain diameter is less than the value calculated based on an equivalent drainage perimeter assumption; based on the finite-element analysis result it has been suggested:

$$d_w = (a + b) / 2 \quad (2)$$

Pradhan et al (1993) suggested that the equivalent diameter of band-shaped drains could be estimated by considering the net flow around a soil cylinder of diameter d_e (Fig 2). The mean square distances of their net flow is calculated as:

$$s^{-2} = \frac{1}{4}d_e^2 + \frac{1}{12}a^2 - \frac{2a}{\pi^2}d_e \text{ then, } d_w = d_e - 2\sqrt{(s^{-2})} + b \quad (3)$$

More recently, Long and Covo (1994) found that the equivalent diameter (d_w) could be computed using an electrical analogue field plotter:

$$d_w = 0.5a + 0.7b \quad (4)$$

The discharge capacity is one of the most important parameters that control the performance of PVDs. The discharge capacity depends primarily on the following factors (Fig. 3): (1) the area of the drain; (2) the effect of lateral earth pressure; (3) possible folding, bending and crimping of the drain, and (4) Infiltration of fine particles into the drain filter (Holtz et al, 1991).

In practice, static and dynamic methods can be used to install PVDs. The static procedure is preferred for driving the mandrel into the ground, whereas the dynamic method can be used to penetrate weathered crust (e.g. drop hammer impact or vibrating hammer). A typical PVD installation in the field is shown in Fig 4. The degree of disturbance during installation depends on the types of soil, the size and shape of the mandrels, and the soil macro-fabric. The sand blanket system is used to expel water away from the drains and provide a sound working mat for erecting the drain installation rigs. Before installing PVDs, surface soil and vegetation must be removed so the site can be graded and a sand blanket placed and compacted. Field instrumentation for monitoring and evaluating embankments is vital to examine and control any geotechnical problems. Instrumentation can be

separated into two categories (Bo et al., 2003) depending on the construction stages; the first is used to prevent sudden failures during construction (e.g. settlement plates, inclinometers and piezometers), whereas the second is related to the rate of settlement and excess pore pressure during loading (e.g. multi-level settlement gauges and piezometers).

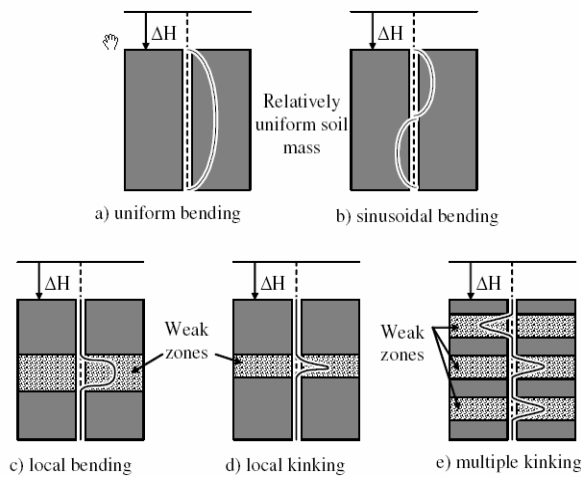


Fig 3. PVD Deformation modes (after Holtz et al., 1991)

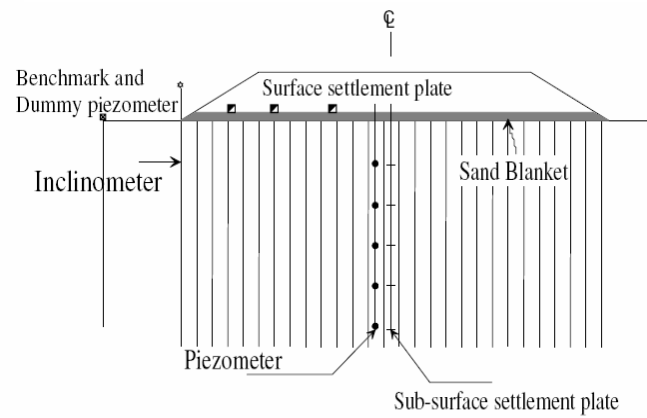


Fig 4. System of PVDs with sand blanket and surcharge preloading (after Indraratna et al., 2005c)

2.2. Factors influencing the drain efficiency

2.2.1. Smear effect

The installation of vertical drains by means of a mandrel causes significant remolding of the subsoil, especially adjacent to the mandrel; a zone of smear will be developed. Soil permeability around the drain can be decreased significantly, which retards the rate of consolidation. This decrease in permeability can be expressed by the ratio of average permeability in the smear zone and the in-situ horizontal permeability (k_r/k_s). This ratio depends on the sensitivity and macro fabric of the soil. The area of the smear zone (d_s) can be influenced by the size and shape of the mandrel, method of installation, and type of soil. These factors can be determined using large-scale consolidation testing (Indraratna and Redana, 1997). The extent of the smear zone can be determined using variations in permeability and water content (Sathanantha and Indraratna, 2006).

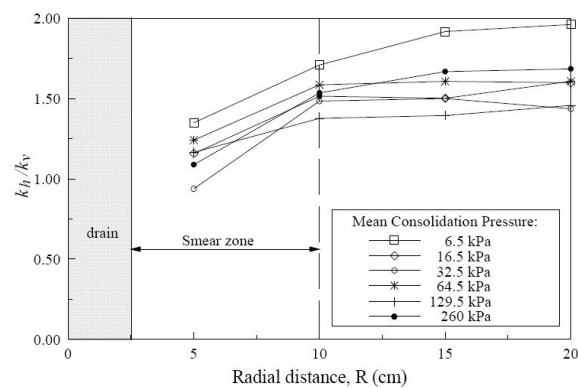


Fig 5. Ratio of k_r/k_v along the radial distance from the central drain (after Indraratna and Redana, 1995)

Based on laboratory tests conducted on a large-scale consolidometer, the extent of the smear zone can be quantified either by variations in permeability or water content along the radial distance (Indraratna and Redana, 1997). Fig. 5 shows the variation of the ratio of the horizontal to vertical permeability (k_r/k_v) at different consolidation pressures along the radial distance, obtained from large-scale laboratory consolidation.

Different relationships have been proposed to determine the size of the smear zone. For design purposes it is proposed that the diameter of the smear zone (d_s) and the cross sectional area of the mandrel:

$$d_s = (2.5 \div 3)d_m \quad (5)$$

Where: (d_m) is the diameter of a circle with an area equal to the cross sectional area of the mandrel or the cross sectional area of the anchor at the tip, which ever is greater. If there are no test data, $d_s = 3d_m$, is suggested (Hansbo 1987).

2.2.2. The effect of drain unsaturation during installation

Unsaturated soil adjacent to the drain can occur due to mandrel withdrawal (air gap) and dry conditions of PVDs. The apparent retardation of pore pressure dissipation and consolidation can be found at the initial stage of loading (Indraratna et al., 2004). Fig 6 shows using a numerical simulation, how the top of the drain takes a longer time to be saturated compared to the bottom of the drain. This example assumes that the PVD was 50% saturated at the start. Even for a short drain such as 1m, the time lag for complete drain saturation can be significant.

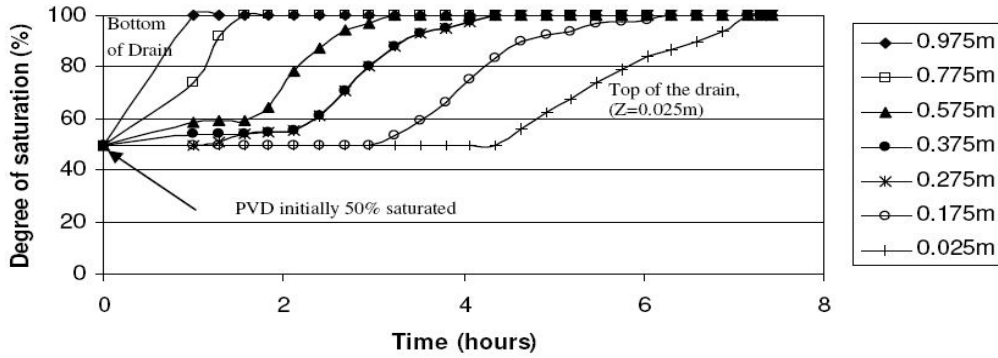


Fig 6. Degree of drain saturation with time (after Indraratna et al. 2004)

3. Theory of Radial consolidation

3.1. Axisymmetric unit cell analysis

3.1.1. Linear Darcian flow

Conventional radial consolidation theory has commonly been used to predict the behavior of vertical drains in soft clay. Its mathematical formulation is based on the small strain theory, where for a given stress range, a constant volume compressibility (m_v) and a constant coefficient of lateral permeability (k_h) are assumed (Barron, 1948; Hansbo, 1981). Indraratna et al. (2005c) have replaced m_v with the compressibility indices (C_c and C_r), which define the slopes of the e - $\log \sigma'$ relationship. Moreover, the variation of horizontal permeability coefficient (k_h) with the void ratio (e) during consolidation is represented by the e - $\log k_h$ relationship which has a slope of C_k .

The main assumptions are given as (Indraratna et al., 2005c): (1) the distribution of vacuum pressure along the boundary of the drain is considered to vary linearly from $-p_0$ at top to $-k_1 p_0$ at the bottom, where k_1 is a ratio between the vacuum at the top and bottom of the drain (Fig 7). (2) The relationship between the average void ratio and logarithm of average effective stress in the normally consolidated range (Fig. 8) can be expressed by $\bar{e} = e_0 - C_c \log(\sigma' / \sigma'_i)$. (3) for radial drainage, the horizontal permeability of soil decreases with the average void ratio (Fig. 9). The relationship between these two parameters is given by Tavenas et al. (1983): $\bar{e} = e_0 + C_k \log(k_h / k_{hi})$. The permeability index (C_k) is considered to be independent of stress history (p'_c), as explained by Nagaraj et al. (1994).

The dissipation rate of average excess pore pressure ratio $R_u = \bar{u}_t / \Delta p$ at any time factor (T_h) can be expressed as:

$$R_u = \left(1 + \frac{p_0 (1 + k_1)}{\Delta p} \right) \exp\left(\frac{-8T_h^*}{\mu} \right) - \frac{p_0 (1 + k_1)}{\Delta p} \quad (6)$$

In the above expression, μ = a group of parameters representing the geometry of the vertical drain system and smear effect. Δp = preloading pressure, T_h is the dimensionless time factor.

$$T_h^* = P_{av} T_h; P_{av} = 0.5 \left[1 + \left(1 + \Delta p / \sigma'_i + p_0 (1 + k_1) / 2\sigma'_i \right)^{-C_c / C_k} \right]; \quad (7)$$

Since the relationship between effective stress and strain is non-linear the average degree of consolidation can be described on either excess pore pressure (stress) (U_p) or strain (U_s). U_p indicates the rate of dissipation of excess pore pressure whereas U_s shows the development rate of surface settlement. Normally, $U_p \neq U_s$ except when the relationship between effective stress and strain is linear, which is in accordance with Terzaghi's one-dimensional theory, and therefore the average degree of consolidation based on excess pore pressure can be obtained as follows:

$$U_p = 1 - R_u \quad (8)$$

The average degree of consolidation based on settlement (ρ) is defined by:

$$U_s = \rho / \rho_\infty \quad (9)$$

Where ρ_∞ is the settlement when $t \rightarrow \infty$.

When the value of C_c / C_k approaches unity and p_0 becomes zero, the solution converges to the conventional solution proposed by Hansbo (1981):

$$R_u = \exp(-8T_h / \mu) \quad (10)$$

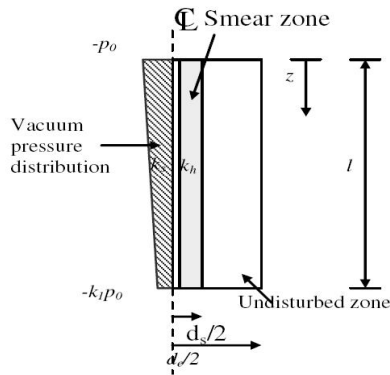


Fig 7. Cylindrical unit cell with linear vacuum pressure distribution (after Indraratna et al., 2005d)

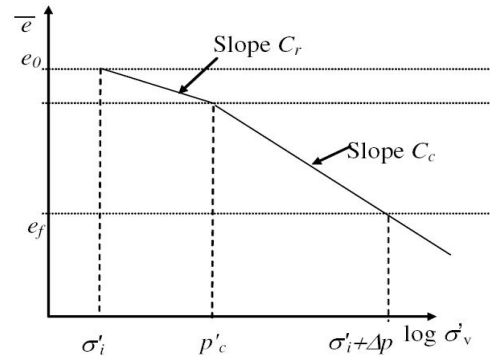


Fig 8. Soil compression curve (after Indraratna et al., 2005c)

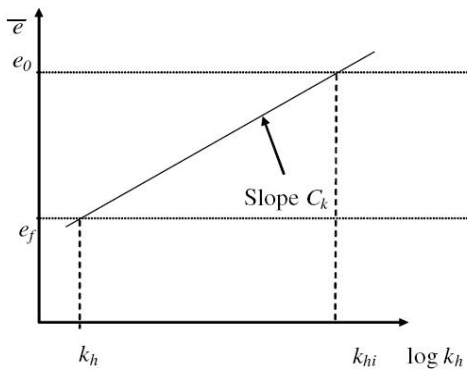


Fig 9. Semi-log permeability-void ratio (after Indraratna et al., 2005c)

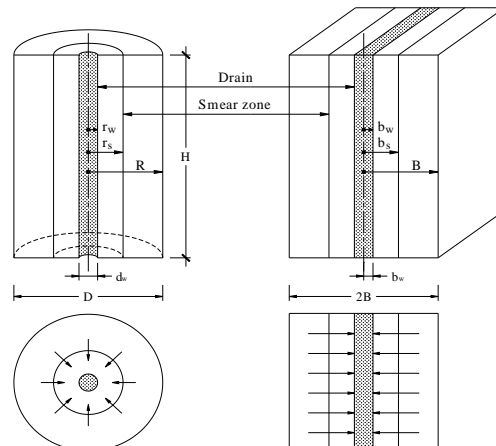


Fig 10. Conversion of an axisymmetric unit cell into plane strain condition (a) Axisymmetric (b) Plane strain

3.1.2. Non-Darcian flow

Hansbo (1997) stated that at small hydraulic gradients, the conventional linear Darcy's law may be replaced by a non-Darcian flow condition defined by an exponential relationship. Based on non-Darcian flow, Hansbo (1997) modified the classical axisymmetric solutions. The pore water flow velocity v caused by a hydraulic gradient i might deviate from the original Darcy's law $v = ki$, where under a certain gradient i_0 below which no flow occurs. Then the rate of flow is given by $v = k(i - i_0)$, hence the following relations have been proposed:

$$v = ki^n \text{ for } i \leq i_1 \quad (11)$$

$$v = k(i - i_0) \text{ for } i \geq i_1 \quad (12)$$

$$\text{Where, } i_1 = i_0 n / (n-1) \text{ and } \kappa = (n^{-1} i_1^{1-n}) k \quad (13)$$

In order to study the non-Darcian effects Hansbo (1979, 1997) proposed an alternative consolidation equation where the time required to attain a certain average degree of consolidation, including the smear effect, is give by:

$$t = \frac{\alpha D^2}{\lambda} \left(\frac{D \gamma_w}{u_0} \right)^{n-1} \left(\frac{1}{(1 - \bar{U}_h)^{n-1}} - 1 \right) \quad (14)$$

Where, the coefficient of consolidation $\lambda = \kappa_h M / \gamma_w$, $M = 1/m_v$ is the oedometer modulus, D is the diameter of the drain influence zone, d_s is the diameter of smear zone, $n = D/d_w$, d_w is the drain diameter, u_0 is the initial average excess pore water pressure, and $\alpha = \frac{n^{2n} \beta^n}{4(n-1)^{n+1}}$ in which.

$$\beta = \frac{1}{3n-1} - \frac{n-1}{n(3n-1)(5n-1)} - \frac{(n-1)^2}{2n^2(5n-1)(7n-1)} + \frac{1}{2n} \left[\left(\frac{\kappa_h}{\kappa_s} - 1 \right) \left(\frac{D}{d_s} \right)^{1-(1/n)} - \frac{\kappa_h}{\kappa_s} \left(\frac{D}{d_w} \right)^{1-(1/n)} \right] \quad (15)$$

3.2. Equivalent plane strain approach for multi-drain analysis

3.2.1. Darcian Flow

Indraratna and Redana (1997, 1998 and 2000) and Indraratna et al. (2005b) converted the vertical drain system shown in Fig. 10 into an equivalent parallel drain wall by adjusting the coefficient of permeability of the soil and by assuming a plane strain cell width of $2B$. The half width of the drain b_w and half width of the smear zone b_s may be kept the same as their axisymmetric radii r_w and r_s , respectively, which suggests $b_w = r_w$ and $b_s = r_s$. Indraratna et al. (2005b) proposed the average degree of consolidation in plane strain condition by:

$$\frac{\bar{u}}{u_0} = \left(1 + \frac{P_{0p}}{u_0} \frac{(1+k_1)}{2} \right) \exp \left(- \frac{8T_{hp}}{\mu_p} \right) - \frac{P_{0p}}{u_0} \frac{(1+k_1)}{2} \quad \text{And, } \mu_p = \left[\alpha + (\beta) \frac{k_{hp}}{k'_{hp}} \right] \quad (16)$$

Where, u_0 =initial excess pore pressure, u =pore pressure at time t (average values), and T_{hp} = time factor in plane strain, k_{hp} and k'_{hp} are the undisturbed horizontal and the corresponding smear zone equivalent permeability, respectively, α and β are geometric parameters α and β calculated by combination of b_s , b_w and B .

At a given effective stress level and at each time step, the average degree of consolidation for both axisymmetric (U_p) and equivalent plane strain ($U_{p,pl}$) conditions are made equal, hence:

$$\bar{U}_p = \bar{U}_{p,pl} \quad (17)$$

Combining Equations (3-5 and (3-8) with equation 14 of original theory by Hansbo, 1981, the time factor ratio can be represented by the following equation:

$$\frac{T_{hp}}{T_h} = \frac{k_{hp}}{k_h} \frac{R^2}{B^2} = \frac{\mu_p}{\mu} \quad (18)$$

Making the magnitudes of R and B to be the same, Indraratna and Redana (2000) presented a relationship between k_{hp} and k_{hp}' . The influence of smear effect can be modeled by the ratio of the smear zone permeability to the undisturbed permeability, as follows:

$$\frac{k_{hp}'}{k_{hp}} = \frac{\beta}{\frac{k_{hp}}{k_h} \left[\ln\left(\frac{n}{s}\right) + \left(\frac{k_h}{k_h'}\right) \ln(s) - 0.75 \right] - \alpha} \quad (19)$$

If smear and well resistance effects are ignored in the above expression, then the simplified ratio of plane strain to axisymmetric permeability is readily obtained, as also proposed earlier by Hird et al. (1992), as follows:

$$\frac{k_{hp}}{k_h} = \frac{0.67}{[\ln(n) - 0.75]} \quad (20)$$

3.2.2. Non-Darcian Flow

Sathananthan and Indraratna (2005) determined a solution for an equivalent plane strain under non-Darcian flow where the converted permeability relationship is given by:

$$\kappa_{hp} = 2\kappa_h \left(\frac{n-1}{2n^2} \frac{\beta_p}{\beta} \right)^n \quad (21)$$

Ignoring the smear effect in Equation (3-16), the equivalent plane strain permeability in the undisturbed zone is now obtained as:

$$\frac{\kappa_{hp}}{\kappa_h} = \frac{\lambda_{hp}}{\lambda} = 2 \left(\frac{f_p(n, b_w / B)}{2f(n, r_w / R)} \right)^n \quad (22)$$

4. Field monitoring and data interpretation

Field monitoring is essential for projects using vacuum preloading as it is the only way to assess the effect of soil improvement using vacuum preloading. Normally, settlements of different soil layers, pore water pressure and lateral displacement at different elevations are measured. The average degree of consolidation (DOC) can be evaluated based on either the settlement or pore pressure data. The DOC estimated using settlement is affected by the methods used for predicting the ultimate settlement. Using the monitored pore water pressure data, the pore water pressure distribution versus depth profiles can be plotted for the initial, final, and any intermediate states. The DOC can be estimated based on the pore water pressure profile using the method suggested by Chu and Yan (2005). One example is shown in Fig 11. As shown by Chu and Yan (2005) using case studies, the DOC estimated using settlement data is generally greater than that using pore water pressure data. This can be partially explained by the fact that when only limited instruments can be used, settlement and pore water pressure gauges will be installed only at the locations where the maximum settlement and pore water pressure will be likely to occur. As a result, the DOC tends to be overestimated when settlement data are used and underestimated when pore water pressure data are used. It has been suggested by Chu and Yan (2005) that for contracting purpose, it is necessary to specify the method used to calculate the DOC and indicate clearly whether the DOC is to be estimated using settlement or pore water pressure data.

5. Numerical modeling

A case study of a combined vacuum and surcharge load through prefabricated vertical drains (PVD) at a storage yard at Tianjin Port, China was investigated using a finite element analysis (Rujikiatkamjorn et al. 20007). At this site, a combination of 80 kPa vacuum pressure and 40 kPa of fill surcharge were required to improve the soft soil condition and avoid any instability problems. Fig. 12 presents soil profile with its relevant soil properties. The vertical cross section and the locations of field instrumentation are shown in Fig 13. This included settlement gauges, pore water pressure transducers, multi-level gauges, inclinometers and piezometers. PVDs of 20m in length (Section: 100 mm x 3 mm) were installed in a square pattern 1m spacing. The finite element mesh (using the finite element software ABAQUS) for plane strain analysis contained elements having 8-node bi-quadratic displacement and bilinear pore pressure shape functions (Fig 14).

Figs 15 and 16 show a comparison between the predicted and recorded field settlements and pore pressure, respectively. The predicted consolidation settlement and pore pressure are in accordance with the measured results. The mean pore pressure is negative (suction), avoiding any potential undrained failures (Indraratna et al., 2005). Figure 17 illustrates the comparison between the measured and predicted lateral movements at the toe of embankment after 180 days. The negative lateral displacement denotes an inward soil movement towards the centerline of the embankment. The predictions at shallow depth (i.e., 0-5m) agree well with the field data, but they slightly underestimate the field results at 5-10m depth (middle of the soft clay layer). It can be seen that vacuum consolidation can minimize any lateral outward yield of soil to increase stability of soft clay foundations.

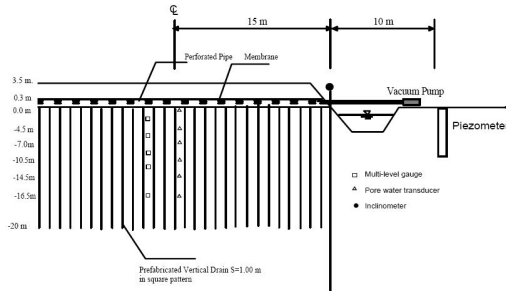


Fig 13. Vertical cross section A-A and locations of monitoring instruments (after Rujikiatkamjorn et al. 2007)

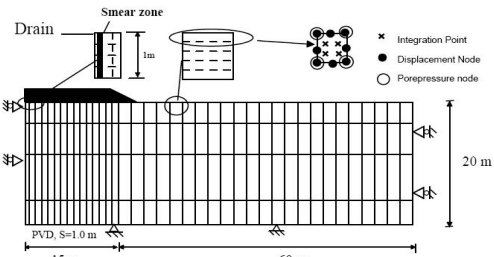


Fig 14. Finite element mesh for plane strain analysis (after Rujikiatkamjorn et al. 2007)

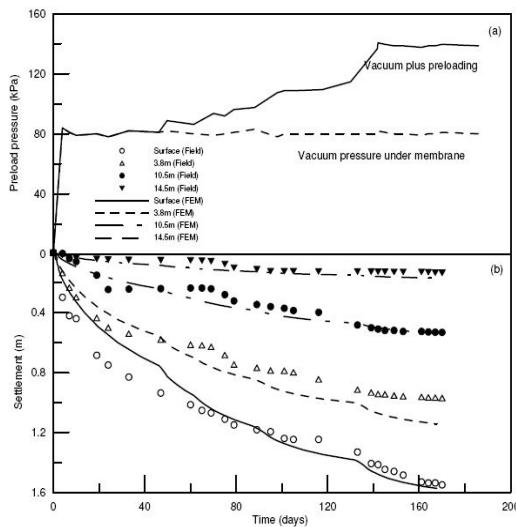


Fig 15. Section II: (a) Loading history and (b) Consolidation settlements (after Rujikiatkamjorn et al. 2007)

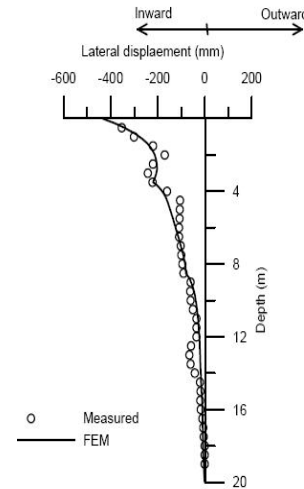


Fig 17. Lateral displacement after 180 days at the embankment toe (after Rujikiatkamjorn et al. 2007)

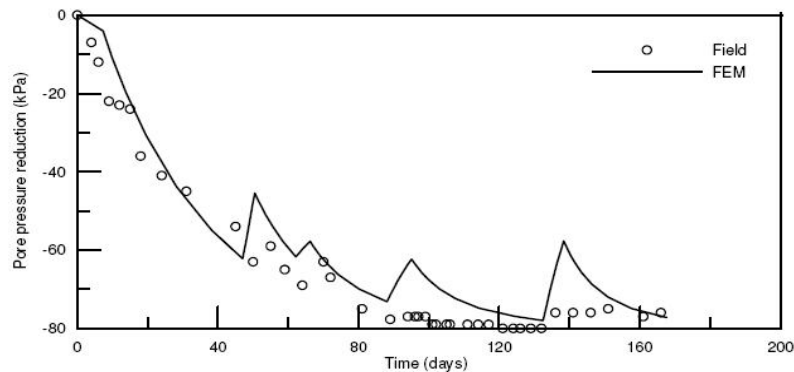


Fig 16. Section II: Pore pressure variation at 0.25m away from the embankment centerline and at 5.5m depth (after Rujikiatkamjorn et al. 2007)

6. Present development

6.1. Use of Drain Panels

Several improvements to the vacuum preloading technique have been made in the recent years. The first is the use of drain panels as shown in Fig. 18, instead of the pipes. This is to ensure the drainage channels will still function well under a high surcharge pressure, as in the case of combined fill and vacuum preloading. The drainage panels also provide better channels for distributing vacuum pressure and water discharging. Some drainage panels also have slots for direction connection with PVDs and thus improve the efficiency of the system.



(a) Corrugated flexible pipes

(b) Other types of geo-composites

Fig 18. Horizontal pipes used for vacuum preloading

6.2. Membrane Free Techniques

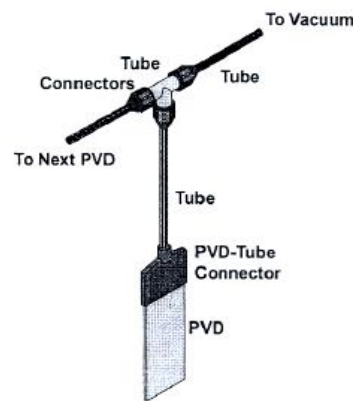


Fig 19. PVD and tubing for vacuum preloading (after Seah, 2006)

When the total area has to be subdivided into a number of sections to facilitate the installation of membrane, the vacuum preloading can only be carried out one section after another. This may not be efficient when the vacuum preloading method is used for land reclamation over a large area. One way to overcome this problem is to connect the vacuum channel directly to each individual drain using a tubing system as shown in Fig. 19. In this way, the channel from the top of the PVD to the vacuum line is sealed. Hence a sand blanket and membranes are not required. This system has been used for the construction of the new Bangkok International Airport (Seah, 2006). However, as such a system does not provide an airtight condition for the entire area, the efficiency of the system can be low. The vacuum pressure applied may only be 50 kPa or lower (Seah, 2006). This method also only works when the soil layer to be improved is dominantly clay with low permeability. Another method to do sway with the membrane is to use the so-called low level vacuum preloading method (Yan and Cao, 2005). This method is schematically illustrated in Fig. 20. When clay slurry is used as fill for land reclamation, the vacuum pipes can be installed at the seabed or a level a few meters below the ground surface. In this way, clay slurry fill can be placed on top of the vacuum pipes. As clay has a low permeability, the fill material will provide a good sealing cap and membranes will be not required. However, this method is not free of problems. Tension cracks will develop in the top layer when dried under the sunlight. It is also difficult to install drainage pipes or panels underwater. Nevertheless, this method does not require the construction of inner dikes for subdivision and thus cuts down the project costs substantially.

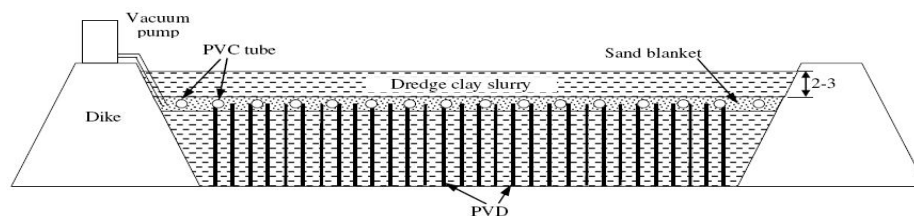


Fig 20. No membrane preloading method

6.3. Dealing with Inter-bedded Permeable Layers

The vacuum preloading method may not work well when the subsoil is inter-bedded with sand lenses or permeable layers that extend beyond the boundary of the area to be improved, such as the improvement of soft soil below sand fill for reclaimed land. In this case, a cut-off wall is required to be installed around the boundary of entire area to be treated. One example is given by Tang and Shang (2000), in which a 120 cm wide and 4.5 m deep clay slurry wall was used as a cut-off wall in order to improve the soft clay below a silty sand layer. However, installation of cut-off walls is expensive when the total area to be treated is large. An alternative method is to use PVD with impermeable plastic sleeve for the section of the PVD that passes through the permeable layer. However, this is workable only when we know fairly accurately the thickness of the permeable layer, which is often the case for reclaimed land.

6.4. Drainage Enhanced Dynamic Compaction Method

One shortcoming of the vacuum preloading or the surcharge preloading method in general is that it is time consuming. One way to overcome this problem is to combine vacuum preloading with dynamic compaction. The basic idea is to use dynamic compaction with low impact energy to generate excess pore pressure which can be then dissipated quickly under the vacuum action. The quick dissipation of pore pressure in turn improves the efficiency of dynamic compaction. This method has been used successfully in a number of projects in China (Xu et al. 2003).

7. Conclusions

An overview on techniques, current theories of radial consolidation, practical aspects and new development of vacuum preloading method are summarized. The main advantage of vacuum application is that the surcharge height of embankments on very soft clays can be reduced to prevent any undrained failure. The review presented in this paper has shown that the vacuum preloading method is still evolving and can be further improved. While, the membrane-based method of vacuum application has been conventional, membrane-less techniques that apply vacuum pressure directly through PVDs are becoming increasingly commercialized. There is still much potential to expand the capacity of the vacuum preloading methods into new applications by combining them with other ground improvement techniques.

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